Consider some typical numbers. The intent will be to provide the best possible numbers that the present device technologies can provide in order to establish a limiting value of S/N. For now the value of ΔC must be derived from experience. A variety of practical consideration limit ΔC. They are package parasitic elements, consideration of resonance conditions which can lead to high losses, and local circuit transformations. It has been the author's experience with a wide variety of oscillator circuits that $\Delta C = 0.5$ pF is the practical upper limit. This number is a peak-to-peak value, converting to rms yields $\Delta C_{\text{rms}} = 0.18$ pF.

The value of $(\partial Cd/\partial V0)$ for Gunn diodes has been found from experiment to be about 0.2 pF/V [8]. This number has been arrived at by two separate experiments. One experiment involved direct observation of the capacity of a stabilized device. The other experiment involved observing the pushing factor of an oscillator, and calculating $(\partial Cd/\partial V0)$ from a knowledge of the circuit Q and the device conductance.

A range of values for $S_{V0}(fm)$ has been given in a previous paper [6] If only the lowest noise devices are considered, an empirical expression for this spectrum in a 3-kHz bandwidth is

$$S_{\Delta V0}(fm)B = \frac{3 \times 10^{-7}}{fm} \ (\text{V}^2).$$

The noise temperature of a low-noise Gunn diode is 30 000 K [6]. Based on a noise figure of 50 dB, the noise temperature of an Si IMPATT is 3×10^7 K. $|-Gd|$ of a Gunn diode is approximately 10^{-2} mho for a 100 mW device.[1] The parallel equivalent circuit of an IMPATT diode also has a $|-Gd|$ of approximately 10^{-2} mho [8].

Fig. 2 shows a comparison of the signal-to-noise ratio of Gunn and IMPATT devices in a 50-mW pump application. As in the amplifier case, 120, 400, 600, 1200, and 1800 channels are considered. Customarily, 70 kHz is the lowest baseband slot. Notice that the Gunn oscillator can just meet the 80-dB specification at 70-kHz baseband for 1800 channels. Under the same conditions the IMPATT oscillator is nearly 30 dB out of spec. Even at low channel capacity the IMPATT oscillator is not close to the 80-dB specification, making IMPATT's look very unattractive for this application. It should be pointed out that since the high noise with Gunn devices occurs at low baseband frequencies, an advantage can be gained by moving the low slot frequency out in baseband.

[1] Unpublished work performed at the Monsanto Company, Microwave Production Group (now a part of Microwave Associates).

IV. Conclusions

1) Gunn amplifiers are capable of handling up to 1800 channels with inputs less than 1 mW.

2) Si IMPATT amplifiers require noiseless preamplification to several hundred milliwatts in order to handle 1800 channels.

3) Gunn oscillators can handle 1800 channels, although barely in the low slot.

4) Si IMPATT oscillators do not appear useful for carrying FM-FDM information.

5) GaAs IMPATT's, with noise properties midway between Si IMPATT's and Gunns may be more practical for 1800 channel amplifier service than Si IMPATT's. GaAs IMPATT's may also be capable of low-capacity oscillator service.

References

[1] Davenport and Root, *Random Signals and Noise.* New York: McGraw-Hill, 1958.
[2] A. A. Sweet, "A study of noise in Gunn effect oscillators," Ph.D. dissertation Cornell Univ., Ithaca, N.Y., 1970.
[3] A. A. Sweet, J. C. Collinet, and R. N. Wallace, "Multistage Gunn amplifiers for FM-CW systems," *IEEE J. Solid-State Circuits,* vol. SC-8, pp. 20–28, Feb. 1973.
[4] C. L. Upadhyayula and B. S. Perlman, "Design and performance of transferred electron amplifiers using distributed equalizer networks," *IEEE J. Solid-State Circuits,* vol. SC-8, pp. 29–36, Feb. 1973.
[5] Thaler, "Avalanche diode circuits," Philadelphia, Pa., ISSCC, 1973.
[6] A. A. Sweet, "A general analysis of noise in Gunn effect oscillators," *Proc. IEEE* (Lett.), vol. 60, pp. 999–1000, Aug. 1972.
[7] M. Ohtomo, "Experimental evaluation of noise parameters in Gunn and avalanche oscillators," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-20, pp. 425–437, July 1972.
[8] "Microwave power generation and amplification using IMPATT diodes," Hewlett-Packard Applications, Note 935.

Letters

Comments on "Wave Propagation on Nonuniform Transmission Lines"

SUHASH C. DUTTA ROY

Abstract—**It is shown that the solutions for wave propagation on a nonuniform transmission line, recently proposed by Bergquist, are alternative forms of, or easily derivable from, the results of Protonotarios and Wing, given earlier.**

I. Introduction

In the above short paper,[1] Bergquist has proposed series solutions for the reflection coefficient, scattering parameters, and the admittance of a general nonuniform transmission line for arbitrary load conditions. The purpose of this letter is to show that these solutions are alternative forms of, or easily derivable from, the results of Protonotarios and Wing, given earlier in [1] and [2].

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[1] A. Bergquist, *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-20, pp. 557–558, Aug. 1972.

II. Protonotarios and Wing Formulas

Protonotarios and Wing formulas, generalized to an arbitrary nonuniform transmission line characterized by a series impedance per unit length $z(x)$ and a shunt admittance per unit length $y(x)$, with notations as in Fig. 1(a), are given by

$$\begin{bmatrix} v(0) \\ i(0) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v(l) \\ i(l) \end{bmatrix} \tag{1}$$

where

$$A = 1 + \int_0^l \int_0^{x_2} z(x_1) y(x_2)\, dx_1\, dx_2 + \int_0^l \int_0^{x_4} \int_0^{x_3} \int_0^{x_2} z(x_1) y(x_2) z(x_3) y(x_4)\, dx_1\, dx_2\, dx_3\, dx_4 + \cdots \tag{2}$$

$$B = \int_0^l z(x)\, dx + \int_0^l \int_0^{x_3} \int_0^{x_2} z(x_1) y(x_2) z(x_3)\, dx_1\, dx_2\, dx_3 + \int_0^l \int_0^{x_5} \int_0^{x_4} \int_0^{x_3} \int_0^{x_2} z(x_1) y(x_2) z(x_3) \cdot y(x_4) z(x_5)\, dx_1\, dx_2\, dx_3\, dx_4\, dx_5 + \cdots \tag{3}$$

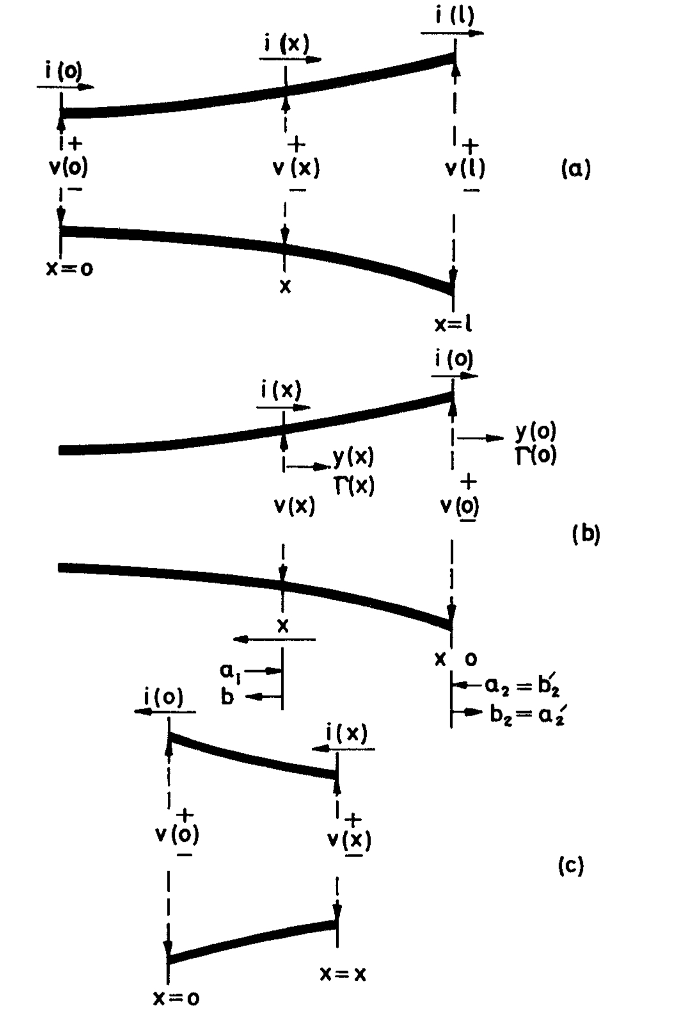

Fig. 1. (a) A section of nonuniform transmission line. (b) Bergquist's notation. (c) A reoriented version of the section of the line in (b) between $x = 0$ and $x = x$.

and

$$C = \text{same as } B \text{ with } z \text{ and } y \text{ interchanged} \quad (4)$$

$$D = \text{same as } A \text{ with } z \text{ and } y \text{ interchanged.} \quad (5)$$

Applied to Bergquist's notations, shown in Fig. 1(b), and considering the section of the line between $x = 0$ and $x = x$, redrawn in Fig. 1(c) for clarity, as a two-port, (1) modifies to

$$\begin{bmatrix} v(0) \\ -i(0) \end{bmatrix} = \begin{bmatrix} A_1(x) & B_1(x) \\ C_1(x) & D_1(x) \end{bmatrix} \begin{bmatrix} v(x) \\ -i(x) \end{bmatrix} \quad (6)$$

where $A_1(x),\cdots,D_1(x)$ are given by (2)–(5), respectively, with l replaced by x. By simple manipulation, one can now find the transmission parameters occurring in

$$\begin{bmatrix} v(x) \\ i(x) \end{bmatrix} = \begin{bmatrix} A(x) & B(x) \\ C(x) & D(x) \end{bmatrix} \begin{bmatrix} v(0) \\ i(0) \end{bmatrix} \quad (7)$$

as $A(x) = D_1(x)$, $B(x) = B_1(x)$, $C(x) = C_1(x)$, and $D(x) = A_1(x)$. Finally, remembering that x_i $(i = 1,2,\cdots)$ are arbitrary dummy variables, one can write

$$A(x) = \sum_{i=0}^{\infty} K_{2i}(x) \qquad B(x) = \sum_{i=0}^{\infty} Q_{2i+1}(x)$$

$$C(x) = \sum_{i=0}^{\infty} K_{2i+1}(x) \qquad D(x) = \sum_{i=0}^{\infty} Q_{2i}(x) \quad (8)$$

where

$$K_0(x) = Q_0(x) = 1$$

$$K_{2i}(x) = \int_0^x z(x)K_{2i-1}(x)\,dx \qquad K_{2i+1}(x) = \int_0 y(x)K_{2i}(x)\,dx$$

$$Q_{2i}(x) = \int_0^x y(x)Q_{2i-1}(x)\,dx \qquad Q_{2i+1}(x) = \int_0^x z(x)Q_{2i}(x)\,dx \quad (9)$$

III. Bergquist's Formulas

The admittance $Y(x)$ in Fig. 1(b) is given by

$$Y(x) \triangleq \frac{i(x)}{v(x)} = \frac{C(x) + Y(0)D(x)}{A(x) + Y(0)B(x)}. \quad (10)$$

Combining this with (9), we get Bergquist's solution for $Y(x)$.

The solution for the reflection coefficient $\Gamma(x)$ can be easily derived from that of $Y(x)$ by noting first that $Y(x)$ satisfies the differential equation

$$Y'(x) + z(x)Y^2(x) = y(x) \quad (11)$$

where the prime denotes differentiation with respect to x. The differential equation for $\Gamma(x)$, as in Bergquist's short paper, $\Gamma'(x) + 2\gamma(x)\Gamma(x) + g(x)\Gamma^2(x) = g(x)$ where $g(x) = \frac{1}{2}[\ln Y_c(x)]'$, $Y_c(x) = [y(x)/z(x)]^{1/2}$ and $\gamma(x) = [y(x)z(x)]^{1/2}$, can be transformed into the same form as (11) by letting $\Gamma(x) = R(x)h(x)$ where

$$h(x) = \exp\left[-2\int_0^x \gamma(x)\,dx\right].$$

The result is

$$R'(x) + [g(x)h(x)]R^2(x) = [g(x)/h(x)]. \quad (12)$$

Hence the solution for $R(x)$ is of the same form as (10) with $Y(0)$ replaced by $R(0) = \Gamma(0)$; $z(x)$ replaced by $g(x)h(x)$; and $y(x)$ replaced by $g(x)/h(x)$ in the calculation of $K_i(x)$ and $Q_i(x)$, $i = 1,2,\cdots$. The solution for $\Gamma(x) = R(x)h(x)$ so obtained is identical with Bergquist's solution.

The formulas for scattering parameters follow from that for $\Gamma(x)$ by applying the definitions, together with the relation $AD - BC = 1$ which holds good for both $Y(x)$ and $R(x)$.

References

[1] E. N. Protonotarios and O. Wing, "Delay and rise time of arbitrarily tapered RC transmission lines," in *1965 IEEE Int. Conv. Rec.*, pt. 7, pp. 1–6.

[2] ——, "Analysis and intrinsic properties of the general nonuniform transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 142–150, Mar. 1967.